

# Module 4 Single-phase AC Circuits

# Lesson 12

## Generation of Sinusoidal Voltage Waveform (AC) and Some Fundamental Concepts

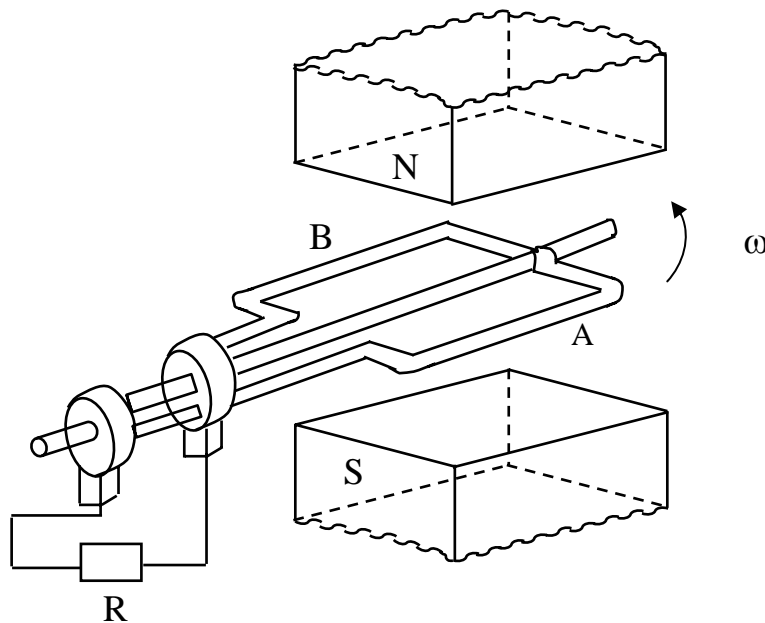
In this lesson, firstly, how a sinusoidal waveform (ac) is generated, is described, and then the terms, such as average and effective (rms) values, related to periodic voltage or current waveforms, are explained. Lastly, some examples to find average and root mean square (rms) values of some periodic waveforms are presented.

Keywords: Sinusoidal waveforms, Generation, Average and RMS values of Waveforms.

After going through this lesson, the students will be able to answer the following questions:

1. What is an ac voltage waveform?
2. How a sinusoidal voltage waveform is generated, with some detail?
3. For periodic voltage or current waveforms, to compute or obtain the average and rms values, and also the time period.
4. To compare the different periodic waveforms, using above values.

## Generation of Sinusoidal (AC) Voltage Waveform



**Fig. 12.1 Schematic diagram for single phase ac generation**

A multi-turn coil is placed inside a magnet with an air gap as shown in Fig. 12.1. The flux lines are from North Pole to South Pole. The coil is rotated at an angular speed,  $\omega = 2\pi n$  (rad/s).

$$n = \frac{\omega}{2\pi} = \text{speed of the coil (rev/sec, or rps)}$$

$$N = 60 \cdot n = \text{speed of the coil (rev/min, or rpm)}$$

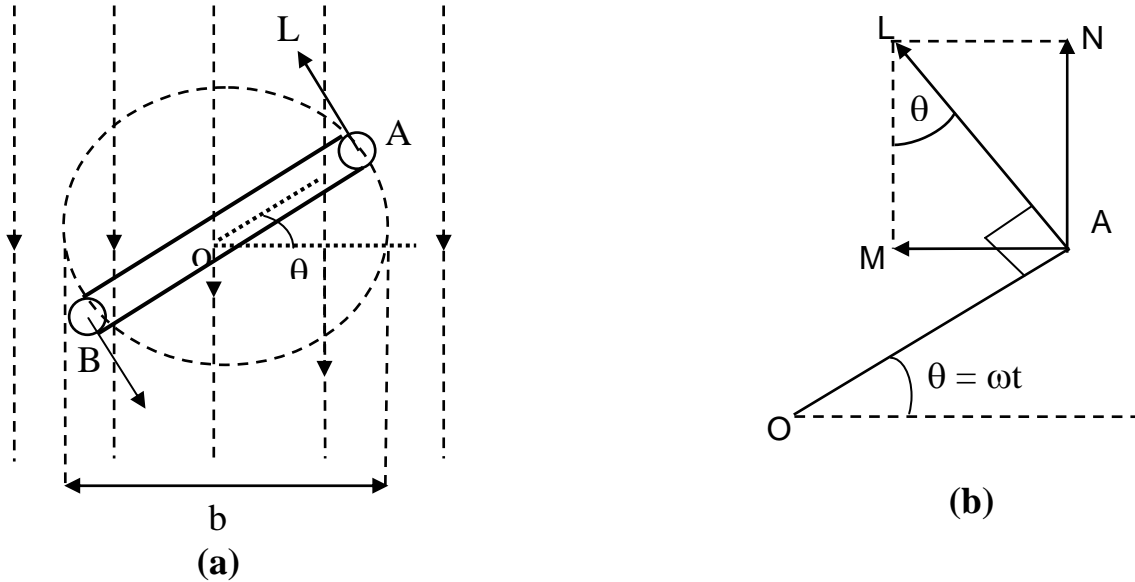
$l$  = length of the coil (m)

$b$  = width (diameter) of the coil (m)

$T$  = No. of turns in the coil

$B$  = flux density in the air gap ( $\text{Wb}/\text{m}^2$ )  
 $v = \pi b n$  = tangential velocity of the coil (m/sec)

### Magnetic Field



**Fig. 12.2 (a) Coil position for Fig. 12.1, and (b) Details**

At a certain instant  $t$ , the coil is at an angle (rad),  $\theta = \omega t$  with the horizontal (Fig. 12.2).

The emf (V) induced on one side of the coil (conductor) is  $B l v \sin \theta$ ,

$\theta$  can also be termed as angular displacement.

The emf induced in the coil (single turn) is  $2 B l v \sin \theta = 2 B l \pi b n \sin \theta$

The total emf induced or generated in the multi-turn coil is

$$e(\theta) = T 2 B l \pi b n \sin \theta = 2 \pi B l b n T \sin \theta = E_m \sin \theta$$

This emf as a function of time, can be expressed as,  $e(t) = E_m \sin \omega t$ . The graph of  $e(t)$  or  $e(\theta)$ , which is a sinusoidal waveform, is shown in Fig. 12.4a

$$\text{Area of the coil (m}^2\text{)} = a = l b$$

$$\text{Flux cut by the coil (Wb)} = \phi = a B = l b B$$

$$\text{Flux linkage (Wb)} = \psi = T \phi = T B l b$$

It may be noted these values of flux  $\phi$  and flux linkage  $\psi$ , are maximum, with the coil being at horizontal position,  $\theta = 0$ . These values change, as the coil moves from the horizontal position (Fig. 12.2). So, also is the value of induced emf as stated earlier.

The maximum value of the induced emf is,

$$E_m = 2 \pi n B l b T = 2 \pi n \phi T = 2 \pi n \psi = \omega \psi = \psi \frac{d\theta}{dt}$$

Determination of frequency (f) in the ac generator

In the above case, the frequency (Hz) of the emf generated is

$f = \omega / (2\pi) = n$ , no. of poles being 2, i.e. having only one pole pair.

In the ac generator, no. of poles =  $p$ , and the speed (rps) =  $n$ , then the frequency in Hz or cycles/sec, is

$f = \text{no. of cycles/sec} = \text{no. of cycles per rev} \times \text{no. of rev per sec} = \text{no. of pairs of poles} \times \text{no. of rev per sec} = (p / 2) n$

$$\text{or, } f = \frac{pN}{120} = \frac{p}{2} \cdot \frac{\omega}{2\pi}$$

### Example

For a 4-pole ac generator to obtain a voltage having a frequency of 50 Hz,

$$\text{the speed is, } n = \frac{2f}{p} = \frac{2 \times 50}{4} = 25 \text{ rps} = 25 \cdot 60 = 1,500 \text{ rpm}$$

For a 2-pole ( $p = 2$ ) machine, the speed should be 3,000 rpm.

Similarly, the speed of the machine having different no. of poles, required to generate a frequency of 50 Hz can be computed.

Sinusoidal voltage waveform having frequency,  $f$  with time period (sec),  $T = 1/f$

## Periodic Voltage or Current Waveform

### Average value

The current waveform shown in Fig. 12.3a, is periodic in nature, with time period,  $T$ . It is positive for first half cycle, while it is negative for second half cycle.

The average value of the waveform,  $i(t)$  is defined as

$$I_{av} = \frac{\text{Area over half cycle}}{\text{Time period of half cycle}} = \frac{1}{T/2} \int_0^{T/2} i(t) dt = \frac{2}{T} \int_0^{T/2} i(t) dt$$

Please note that, in this case, only half cycle, or half of the time period, is to be used for computing the average value, as the average value of the waveform over full cycle is zero (0).

If the half time period ( $T/2$ ) is divided into 6 equal time intervals ( $\Delta T$ ),

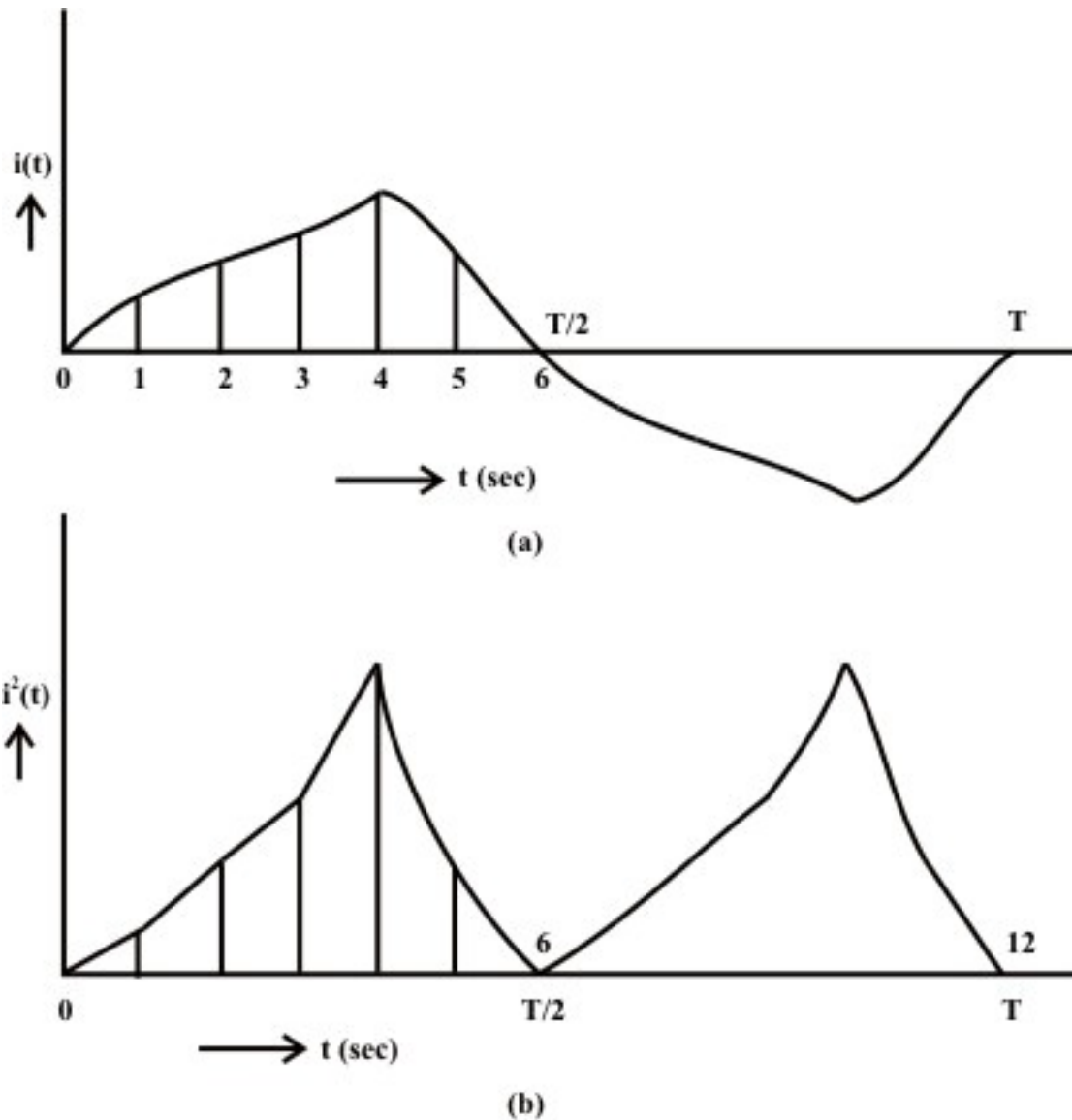
$$I_{av} = \frac{(i_1 + i_2 + i_3 + \dots + i_6) \Delta T}{6 \cdot \Delta T} = \frac{(i_1 + i_2 + i_3 + \dots + i_6)}{6} = \frac{\text{Area over half cycle}}{\text{Time period of half cycle}}$$

Please note that no. of time intervals is  $n = 6$ .

### Root Mean Square (RMS) value

For this current in half time period subdivided into 6 time intervals as given above, in the resistance  $R$ , the average value of energy dissipated is given by

$$\propto \left[ \frac{(i_1^2 + i_2^2 + i_3^2 + \dots + i_6^2)}{6} \right] R$$



**Fig. 12.3 Periodic current waveform**  
**(a) Current (i), (b) Square of current ( $i^2$ )**

The graph of the square of the current waveform,  $i^2(t)$  is shown in Fig. 12.3b. Let  $I$  be the value of the direct current that produces the same energy dissipated in the resistance  $R$ , as produced by the periodic waveform with half time period subdivided into  $n$  time intervals,

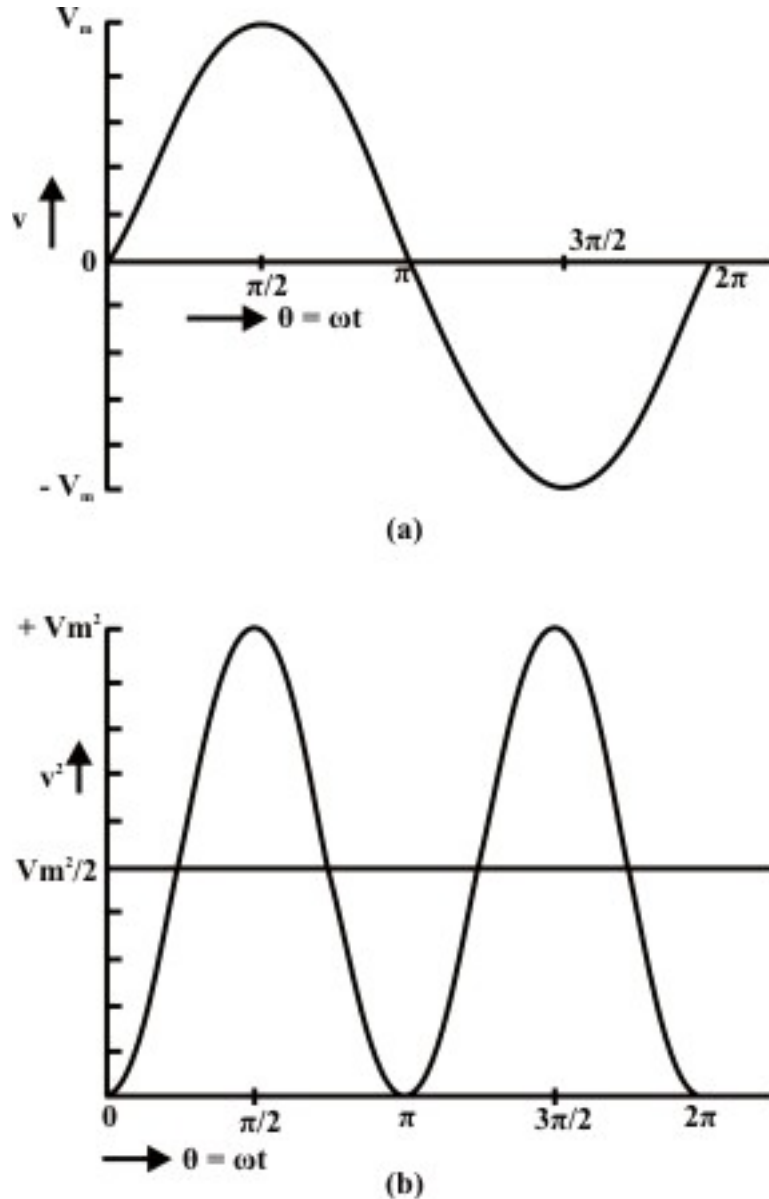
$$I^2 R = \left[ \frac{(i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2) \Delta T}{n \cdot \Delta T} \right] R$$

$$I = \sqrt{\frac{(i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2) \Delta T}{n \cdot \Delta T}} = \sqrt{\frac{\text{Area of } i^2 \text{ curve over half cycle}}{\text{Time period of half cycle}}}$$

$$= \sqrt{\frac{1}{T/2} \int_0^{T/2} i^2 dt} = \sqrt{\frac{2}{T} \int_0^{T/2} i^2 dt}$$

This value is termed as Root Mean Square (RMS) or effective one. Also to be noted that the same rms value of the current is obtained using the full cycle, or the time period.

## Average and RMS Values of Sinusoidal Voltage Waveform



**Fig. 12.4 Sinusoidal voltage waveform  
(a) Voltage (v), (b) Square of voltage (v<sup>2</sup>)**

As shown earlier, normally the voltage generated, which is also transmitted and then distributed to the consumer, is the sinusoidal waveform with a frequency of 50 Hz in

this country. The waveform of the voltage  $v(t)$ , and the square of waveform,  $v^2(t)$ , are shown in figures 12.4a and 12.4b respectively.

Time period,  $T = 1/f = (2\pi)/\omega$ ; in angle ( $\omega T = 2\pi$ )

Half time period,  $T/2 = 1/(2f) = \pi/\omega$ ; in angle ( $\omega T/2 = \pi$ )

$v(\theta) = V_m \sin \theta$  for  $\pi \leq \theta \leq 0$ ;  $v(t) = V_m \sin \omega t$  for  $(\pi/\omega) \leq t \leq 0$

$$V_{av} = \frac{1}{\pi} \int_0^\pi v(\theta) d\theta = \frac{1}{\pi} \int_0^\pi V_m \sin \theta d\theta = \frac{V_m}{\pi} \cos \theta \Big|_0^\pi = \frac{2}{\pi} V_m = 0.637 V_m$$

$$V = \left[ \frac{1}{\pi} \int_0^\pi v^2 d\theta \right]^{\frac{1}{2}} = \left[ \frac{1}{\pi} \int_0^\pi V_m^2 \sin^2 \theta d\theta \right]^{\frac{1}{2}} = \left[ \frac{V_m^2}{\pi} \int_0^\pi \frac{1}{2} (1 - \cos 2\theta) d\theta \right]^{\frac{1}{2}}$$

$$= \left[ \frac{V_m^2}{2\pi} \left( \theta - \frac{1}{2} \sin 2\theta \right) \Big|_0^\pi \right]^{\frac{1}{2}} = \left[ \frac{V_m^2}{2\pi} \pi \right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

or,  $V_m = \sqrt{2} V$

If time  $t$ , is used as a variable, instead of angle  $\theta$ ,

$$V_{av} = \frac{1}{\pi/\omega} \int_0^{\pi/\omega} v(t) dt = \frac{\omega}{\pi} \int_0^{\pi/\omega} V_m \sin \omega t dt = \frac{\omega V_m}{\pi \omega} \cos (\omega t) \Big|_0^{\pi/\omega} = \frac{2}{\pi} V_m = 0.637 V_m$$

In the same way, the rms value,  $V$  can be determined.

If the average value of the above waveform is computed over total time period  $T$ , it comes out as zero, as the area of first (positive) half cycle is the same as that of second (negative) half cycle. However, the rms value remains same, if it is computed over total time period.

The different factors are defined as:

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.707 V_m}{0.637 V_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{Average value}} = \frac{V_m}{0.707 V_m} = 1.414$$

*Note:* The rms value is always greater than the average value, except for a rectangular waveform, in which case the heating effect remains constant, so that the average and the rms values are same.

### Example

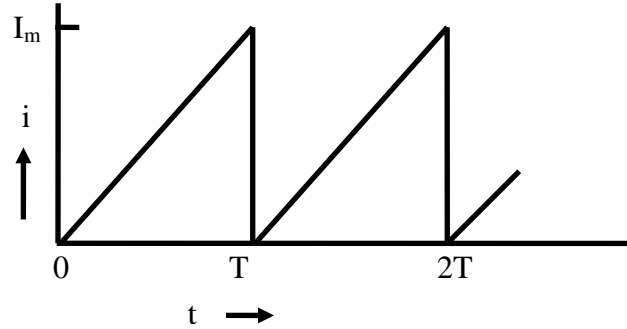
The examples of the two waveforms given are periodic in nature.

1. Triangular current waveform (Fig. 12.5)

Time period =  $T$

$$i(t) = I_m \frac{t}{T} \text{ for } T \leq t \leq 0$$





**Fig. 12.5 Triangular current waveform**

$$I_{av} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \int_0^T I_m \frac{t}{T} dt = \frac{I_m}{T^2} \left[ \frac{t^2}{2} \right]_0^T = \frac{I_m}{T^2} \frac{T^2}{2} = \frac{I_m}{2} = 0.5 I_m$$

$$I = \left[ \frac{1}{T} \int_0^T i^2 dt \right]^{\frac{1}{2}} = \left[ \frac{1}{T} \int_0^T I_m^2 \frac{t^2}{T^2} dt \right]^{\frac{1}{2}} = \left[ \frac{I_m^2}{T^3} \cdot \left[ \frac{t^3}{3} \right]_0^T \right]^{\frac{1}{2}} = \left[ \frac{I_m^2}{T^3} \cdot \frac{T^3}{3} \right]^{\frac{1}{2}} = \left[ \frac{I_m^2}{3} \right]^{\frac{1}{2}}$$

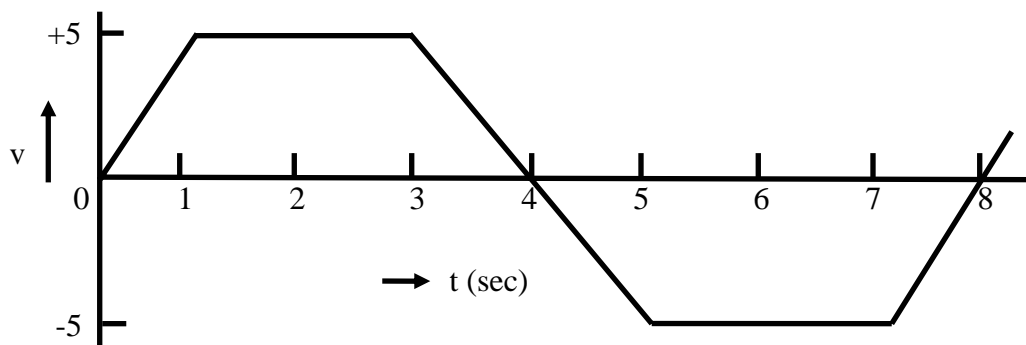
$$= \frac{I_m}{\sqrt{3}} = 0.57735 I_m$$

Two factors of the waveform are:

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.57735 I_m}{0.5 I_m} = 1.1547$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{Average value}} = \frac{I_m}{0.5 I_m} = 2.0$$

To note that the form factor is slightly higher than that for the sinusoidal waveform, while the peak factor is much higher.



**Fig. 12.6 Trapezoidal voltage waveform**

2. Trapezoidal voltage waveform (Fig. 12.6)

Time period (T) = 8 ms

Half time period ( $T/2$ ) =  $8/2 = 4$  ms

$$v(t) = mt = (5/1)t = 5t \text{ for } 1 \leq t \leq 3; \quad v(t) = 5 \text{ for } 3 \leq t \leq 4;$$

$$v(t) = 5(4-t) \text{ for } 4 \leq t \leq 5$$

Please note that time,  $t$  is in ms, and slope,  $m$  is in V/ms. Also to be noted that, as in the case of sinusoidal waveform, only half time period is taken here for the computation of the average and rms values.

$$V_{av} = \frac{1}{T/2} \int_0^{T/2} v(t) dt = \frac{1}{4} \left[ \int_0^1 5t dt + \int_1^3 5 dt + \int_3^4 5(4-t) dt \right] = \frac{1}{4} \left[ \frac{5}{2} t^2 \Big|_0^1 + 5t \Big|_1^3 + \frac{5}{2} (4-t)^2 \Big|_3^4 \right]$$

$$= \frac{1}{4} \left[ \frac{5}{2} + 5(3-1) + \frac{5}{2} \right] = \frac{15}{4} = 3.75 \text{ V}$$

$$V = \left[ \frac{1}{T/2} \int_0^{T/2} v^2 dt \right]^{\frac{1}{2}} = \left[ \frac{1}{4} \left( \int_0^1 (5t)^2 dt + \int_1^3 (5)^2 dt + \int_3^4 5(4-t)^2 dt \right) \right]^{\frac{1}{2}}$$

$$= \left[ \frac{1}{4} \left( \frac{25}{3} t^3 \Big|_0^1 + 25t \Big|_1^3 + \frac{25}{3} (4-t)^3 \Big|_3^4 \right) \right]^{\frac{1}{2}} = \left[ \frac{1}{4} \left( \frac{25}{3} + 25(3-1) + \frac{25}{3} \right) \right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{50}{3}} = \sqrt{16.67} = 4.0825 \text{ V}$$

Two factors of the waveform are:

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{4.0825}{3.75} = 1.0887$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{Average value}} = \frac{5.0}{3.75} = 1.3333$$

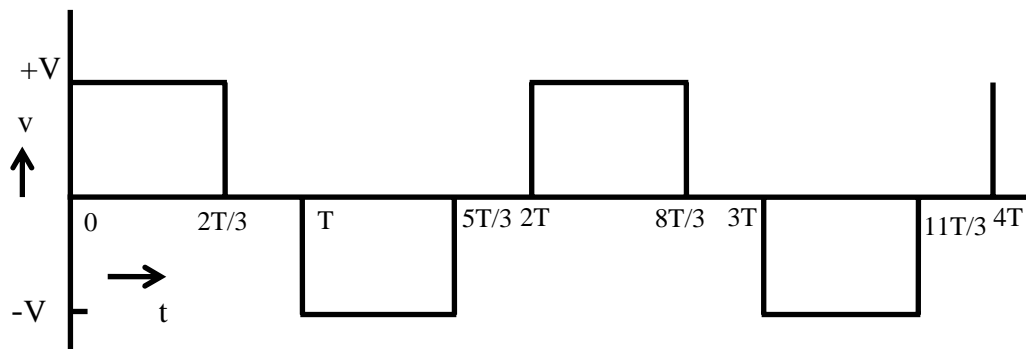
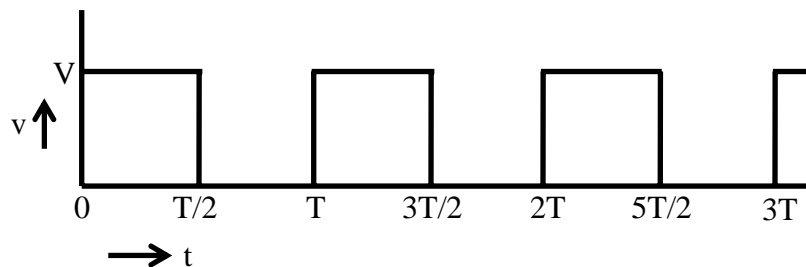
To note that the both the above factors are slightly lower than those for the sinusoidal waveform.

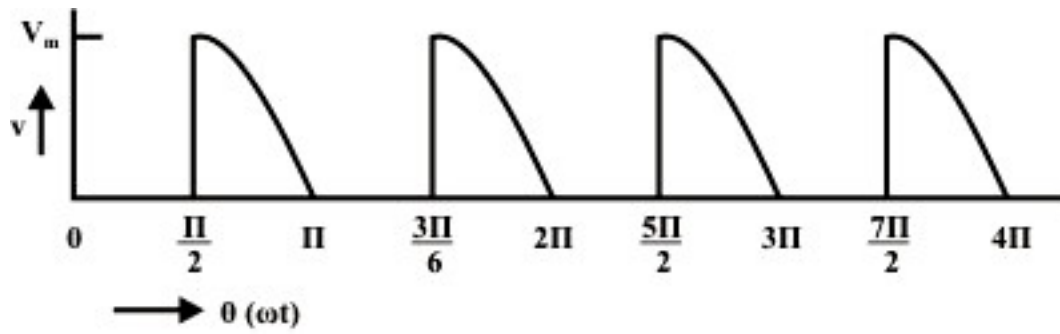
Similarly, the average and rms or effective values of periodic voltage or current waveforms can be computed.

In this lesson, starting with the generation of single phase ac voltage, the terms, such as average and rms values, related to periodic voltage and current waveforms are explained with examples. In the next lesson, the background material required – the representation of sinusoidal voltage/current as phasors, the rectangular and polar forms of the phasors, as complex quantity, and the mathematical operations – addition/subtraction and multiplication/division, using phasors as complex quantity, are discussed in detail with numerical examples. In the following lessons, the study of circuits fed from single phase ac supply, is presented.

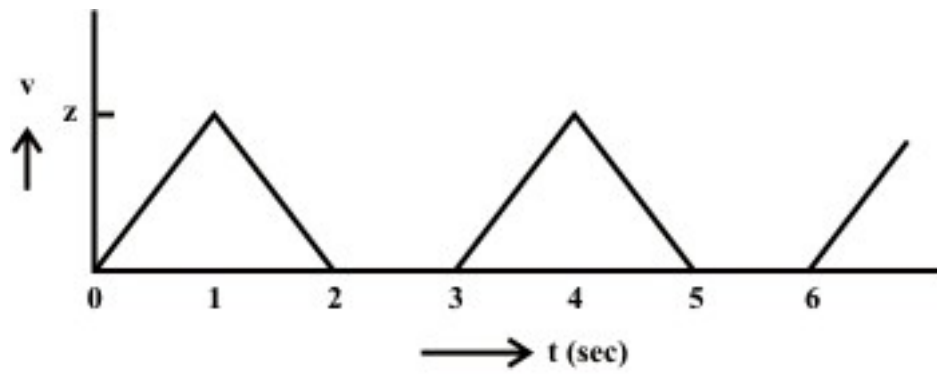
## Problems

- 12.1 What is the speed in rpm of an ac generator with 4 poles, to produce a voltage with a frequency of 50 Hz
- (a) 3000                      (b) 1500                      (c) 1000                      (d) 750
- 12.2 Determine the No. of poles required in an ac generator running at 1,000 rpm, to produce a voltage with a frequency of 50 Hz.
- (a) 2                      (b) 4                      (c) 6                      (d) 8
- 12.3 Calculate the speed in rpm of an ac generator with 24 poles, to produce a voltage with a frequency of 50 Hz.
- (a) 300                      (b) 250                      (c) 200                      (d) 150
- 12.4 Determine the average and root mean square (rms) values of the following waveforms.





(c)



(d)

Fig. 12.7

## List of Figures

Fig. 12.1 Schematic diagram for single phase ac generation

Fig. 12.2 (a) Coil position for Fig. 12.1, and (b) Details

Fig. 12.3 Periodic current (i) waveform

(a) Current (i), (b) Square of current ( $i^2$ )

Fig. 12.4 Sinusoidal voltage waveform

(a) Voltage (v), (b) Square of voltage ( $v^2$ )

Fig. 12.5 Triangular current waveform

Fig. 12.6 Trapezoidal voltage waveform